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Combined Convection of Packer Fluid Flow between Vertical Parallel Plates

by

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APPROVED, THESIS COMMITTEE

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Abstract

Packer fluid whose function is to prevent or tremendously reduce the heat transfer rate which would occur from the production tubing area to the production casing region is being studied in recent decades because of its wide applications in Oil & Gas Industry. Reduction of heat transfer rate can lead to the minimization of trapped annular pressures and reduction of contents of hydrates resolvable in production fluids. This paper utilizing ANSYS Fluent gives numerical solution for the combined convection problem of this packer fluid. Because of the geometry of the tubing-to-casing annulus, it is modeled as vertical and long parallel plates in ANSYS Fluent geometry part where the width of the duct is small comparable to the length of the duct. The flow is assumed to lie in laminar region and ANSYS Fluent laminar flow model is utilized. How different parameters including aspect ratio, temperature difference and inlet velocity will have effects on the convective heat transfer rate are analyzed respectively by measuring and calculating dimensionless parameters including Nusselt number, Reynolds number, Prandtl number, Grashof number and Rayleigh number. Numerical results characterize the convective heat transfer performance of the packer fluid.

Acknowledgement

As time goes by, finally my graduate study has come to its end. It is the time that I formally wave back to my two years' graduate study at Rice University and look forward to future study in the future.

First of all, I must be grateful for my advisor Dr. Yildiz Bayazitoglu. I will never forget the first moment that I met with her in her office. Her kindness made me feel really comfortable just like talking with my own grandmother. Without her supervision and guidance, I cannot handle the research work with such high quality. Sometimes during the group meeting, I misunderstood her points and worked on an opposite direction as she required, however, she was always patient and eager to help me out of the troubles. Thank you for all the time and energy you spent talking with me and giving me recommendations. She is not only an advisor for my academic path, but also a supervisor for my life. During the first meeting, she told me how to be a successful international student once studying abroad for the first time. It was her encouragements that gave me lots of impetus for my graduate study. During her class in the second semester, she not only taught us knowledge in heat transfer area based on her profound knowledge background, but also showed us the way of how to treat and solve engineering problems in a reasonable way. I am confident that what I gained through these two years' study will definitely be helpful for my future study.

In the second place, I must be thankful for Rice University. Without being admitted, I will never have the experience of studying in such a wonderful atmosphere and meeting people with different cultural background from all over the world. These learning experiences at Rice University will always be my precious wealth in my life. I am really grateful for all the professors who taught me lessons, for all the colleagues who helped me figure out academic problems, and for all the friends who assisted me get out of troubles. No matter how far I am away from here, how many achievements I will earn in my future life, I will never forget and always be proud that I am a member of Rice University.

In the end, I really appreciate the attendance of Dr. Yildiz Bayazitoglu, Dr. Enrique Barrera and Dr. Pedram Hassanzadeh to be my committee member for my final thesis

defense. I really appreciate your recommendations during the defense. In addition, through your questions and suggestions, I have a better prospective of the future work.

Nomenclature

d	spacing between duct walls	D_h	hydraulic diameter
h	convective heat transfer coefficient	β	thermal expansion coefficient
μ	apparent viscosity	ν	kinematic viscosity
k	thermal conductivity	ρ	density
y	axial Coordinate	x	transverse coordinate
Y	dimensionless axial coordinate	X	dimensionless transverse coordinate
u	axial velocity	v	transverse velocity
u_c	centerline axial velocity	u_0	inlet velocity
U	dimensionless axial velocity	V	dimensionless transverse velocity
U_c	dimensionless centerline velocity u_c/u_0	r_T	wall temperature difference ratio
r_H	wall heat flux ratio q_1/q_2	Pr	Prandtl number
Ra	Rayleigh number	Gr	Grashof number
Re	Reynolds number	Ar	Archimedes number
Nu	Nusselt number	T	temperature
P	dimensionless pressure difference	θ	dimensionless temperature difference
T_0	inlet temperature	T_1	hot wall temperature
T_2	cold wall temperature	T_m	fluid temperature
q_1	heat flux at hot wall	q_2	heat flux at cold wall

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1. Introduction

1.1 History of combined convection of Newtonian fluid

At the beginning, researchers spent time on the study of forced convection and free convection respectively. However, during the process of forced convection, gravitational forces cannot be neglected in some conditions. Within this condition, forced convection problem becomes combined convection problem. For this reason, researchers transferred the academic direction to combined convection problem. In the past, majority of research studies focused on combined convection problem of flow in circular tube geometry. Both exact solutions and experimental observations for buoyancy-assisted fully-developed laminar flows were provided by Hanratty et al. [1]. Through the observation and experiments in vertical circular tube, existence of reverse flow was found at low Reynolds number. Since that time, combined convection problem of fluid flowing in ducts with different geometries were studied by researchers respectively. Iqbal et al. [2,3] studied combined convection problem of fluid flow through vertical rectangular shape ducts and non-circular ducts in two papers respectively. Afterwards, Kim [4], Maitra and Raju [5] obtained analytical solutions of laminar fully-developed flow in vertical concentric circular annulus under isoflux wall boundary condition.

However, as time went on, more research began to focus on combined convection of fluid flow between parallel plate ducts because this geometry became more and more involved in engineering fields such as modern electronic equipment, nuclear reactors, solar systems and heat exchangers. Tao [6] offered an analytical method of solving fully developed mixed convection problem in a vertical parallel-plate duct. Habchi and Acharya [7] provided a numerical solution of mixed convection of air in a heated vertical channel under either symmetric heating or asymmetric heating. Aung and Worku [8,9] gave out theoretical solutions for combined convection of laminar fluid flow within vertical parallel plates under both symmetric and asymmetric heating boundary conditions. In Aung and Worku's papers, when reverse flow would be detected was also shown. Aung and Worku [10] also solved combined convection problem between vertical parallel plates with both symmetric and asymmetric wall heat fluxes boundary condition.

Cheng et al. [11] made an improvement of combined convection based on the previous literatures and provided a paper which concerned flow and heat transfer characteristics of combined convection of laminar fully developed fluid flow in vertical channels under combinations of thermal boundary conditions. Later, Barletta [12] solved the combined convection problem of fully developed fluid flowing in a vertical parallel plate duct with constant wall temperature boundary condition by taking into account viscous dissipation effect. Recently, Galanis and Behzadmehr [13] presented a review containing experimental, analytical and numerical results of the research on mixed convection of Newtonian fluid in vertical ducts during the past years.

1.2 History of combined convection of power law fluid

Many of the fluids used as industrial purposes are non-Newtonian fluid types such as glues and paints. Their flow and heat transfer behaviors are totally different from Newtonian fluids. So only considering convection problem of Newtonian fluid is not enough. More and more research concentrating on convection problem of non-Newtonian fluid just started from recent decades. Gao and Hartnett [14], Capobianchi and Irvine [15] provided a fully developed forced convection of power-law fluid in rectangular ducts and annular ducts respectively. Later, Olek [16] came up with an analytical method of solving the temperature field of thermal entrance region of fluid flowing within either a circular or parallel plate duct. Patel and Ingham [17] presented analytic solution for the combined convection problem of limiting case of power-law fluids in a vertical parallel plate duct within asymmetric uniform temperature boundary condition. Ingham and Jones [18,19] utilizing finite different method to study the effects of the existence of buoyancy forces in the entrance region of a vertical parallel plate channel and provided a numerical solution for that specific problem. Lin and Hsu [20] solved non-Newtonian fluids flowing between vertical parallel plates with a fully developed velocity profile. Etemad et al. [21] performed a numerical investigation as an extensive research of the previous work on Newtonian fluid to predict the characteristics of the simultaneously developing laminar flow and heat transfer of non-Newtonian fluid.

1.3 Purpose of this paper

The purpose of this paper is to analyze the combined convection of packer fluid flowing between vertical parallel plates. The motivation to solve this problem is to prevent or drastically reduce heat transfer from production fluid to the outer casing region. This type of heat transfer will cause the heating and large pressures buildup of fluids in outer annuli which will lead to the loss and collapse of the entire well. At the same time, gas hydrates, paraffin deposits and asphalt which are resolvable in the fluid will precipitate into the production fluid because of the heat loss. To prevent these kinds of problems from happening, characteristics of packer fluid should be analyzed comprehensively. Bayazitoglu et al. [22] presented a paper of modeling packer fluid as Bingham fluid type. However, for this paper, packer fluid will be modeled as a type of power-law fluid with the behavior index n=0.5. The two plates are kept with constant but different wall temperatures. The numerical results are obtained by solving the governing equations for this specific problem by the code embedded in ANSYS Fluent software. Conclusions are made based on analysis of dimensionless parameters including Reynolds number, Grashof number, Rayleigh number, Prandtl number and Nusselt number. The objective is to gain a quantitative understanding of the mixed convection problem of this widely used packer fluid in Oil & Gas Industry. The analysis of the relationship between fluid properties and heat transfer correlation will enable the better design of packer fluid.

2. Verification of the model

2.1 Fluent introduction

As mentioned in Introduction section, the purpose of the present paper is to analyze the combined convection problem of packer fluid between vertical parallel plates. ANSYS Fluent which is a commercial package for computational fluid dynamics (CFD) is utilized to solve the problems numerically. ANSYS Fluent has become one of the most powerful computational fluid dynamics (CFD) software available at the moment. Its high efficiency, stability and reliability when compared with other software enable the customer to optimize the product's performance in a quick rate. Varieties of physical models and different kinds of numerical solution methods embodied into ANSYS Fluent can definitely help customers build up models in any engineering fields related to flow, heat transfer and reactions.

There are mainly four steps for ANSYS Fluent to get the simulation result including modeling, meshing, setup and running the calculation. Once one step is finished, a checked symbol will appear afterwards which means that the customer can go into the next step. Only if all the four steps are finalized, the convergent solution can be obtained and the results can be viewed in different ways such as contours, path lines and vectors in the solution and results part respectively. A model with all checked symbols is listed below as an example.

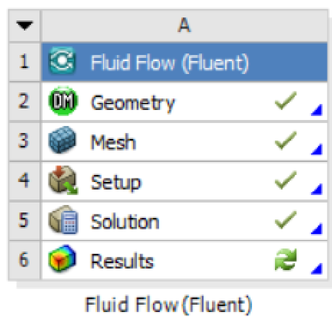

Fig. 1 ANSYS Fluent sample.

2.2 Verification of Newtonian fluid

Before using the model established in ANSYS Fluent to solve the problem, the model must be checked to be precise. Otherwise, the result will not be accurate and reliable. Some existing results are used as comparison to check the ANSYS Fluent model. Newtonian fluid and non-Newtonian fluid will be checked respectively in the later sections in this chapter.

This present paper concentrates on convection problem of packer fluid between vertical parallel plates. The 2-D parallel plates should be established with fixed dimensions of its width and length in Geometry part. The model utilized in this section for verification and the model utilized in later sections for calculating the problem are the same but the dimensions may not be the same. The model is relatively easy to build up and then comes to the mesh. The quality of mesh will determine how accurate the results could be. The finer the mesh is the higher quality the results will have. Once the dimensions of the model are changed, the mesh size should change accordingly to guarantee the accuracy. A pretty good mesh for the given dimensions of verification problem is displayed as below. The verification will be divided into two parts in the following subsections, one with constant wall temperature boundary condition and one with constant wall heat flux boundary condition but with the same geometry, mesh and physical properties.

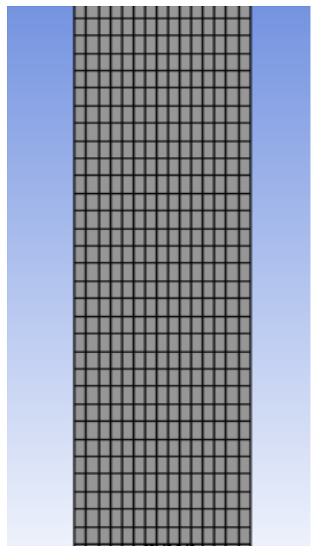

Fig. 2 Mesh sample for verification problem.

2.2.1 Constant wall temperature verification

The boundary condition of the problem analyzed in this paper is constant wall temperature, so constant wall temperature boundary condition is checked first. Win Aung and G. Worku [9] presented a good reference for Newtonian fluid flow between vertical parallel plates with constant wall temperature boundary condition. Both symmetric wall temperature and asymmetric wall temperature boundary condition are taken into consider in this reference paper and will be compared respectively with ANSYS Fluent results. The results obtained directly from ANSYS Fluent are dimensional, so the results should be post-processed and non-dimensionalized by MATLAB and then compared with the given figures in the reference paper. To check the accuracy of ANSYS Fluent model, the same physical parameters should be employed here for ANSYS Fluent to obtain the results. The literature demonstrates that the Prandtl number of air at the working temperature is 0.72. From the air properties data provided online, the physical properties are chosen at the temperature of 250 Kelvin for calculation. The physical properties including density, thermal conductivity, dynamic viscosity, specific heat and thermal expansion coefficient value are listed in Table 1. In addition, the same parameter values will be utilized within the later subsection for constant wall heat flux boundary condition verification.

Table 1 Physical properties of air

Density	$1.426 kg / m^3$
Thermal Conductivity	$0.022268 W / m \cdot K$
Dynamic Viscosity	$1.6068 \times 10^{-5} kg / m \cdot s$
Specific Heat	$1005.4 J / kg \cdot K$
Thermal Expansion Coefficient	$0.0039976 / K$

Firstly, symmetric wall temperature boundary condition is considered. The variable r_T which demonstrates the ratio of wall temperature difference is seen as the judgment of symmetric or asymmetric wall temperature boundary condition. Once this parameter is equal to one, the two walls will have the same wall temperature or the so called symmetric temperature boundary condition. Otherwise, it indicates that the two walls will have different but constant wall temperature.

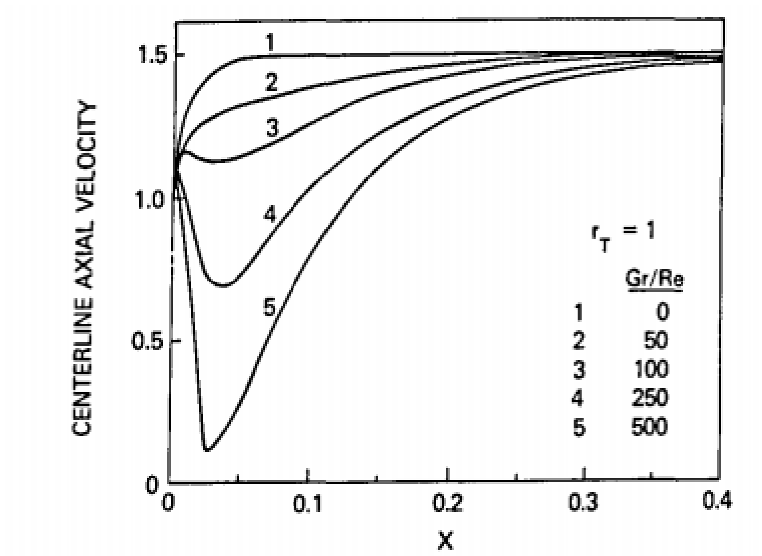

Fig. 3 Centerline velocity for symmetric heating [9].

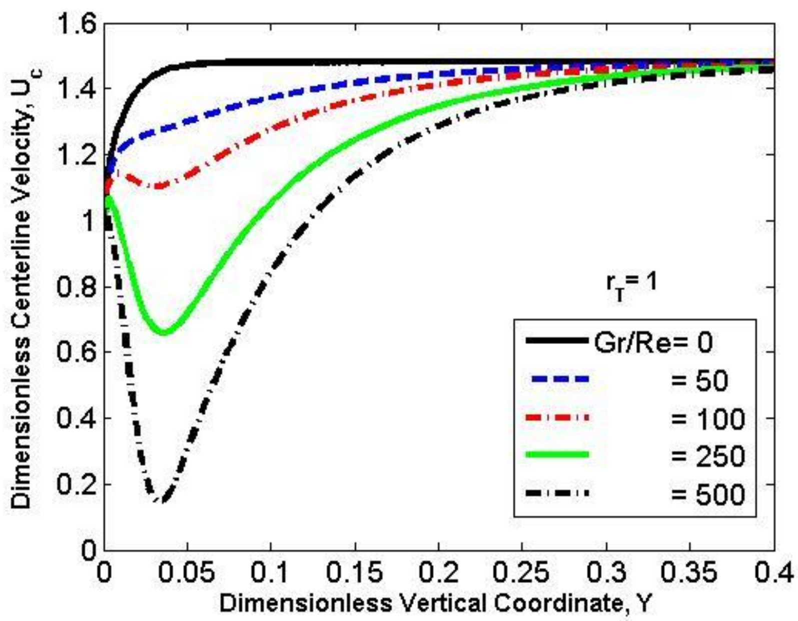

Fig. 4 Centerline velocity for symmetric heating.

For the comparison between Fig. 3 and Fig. 4, symmetric wall heating boundary condition is employed as mentioned. Both figures demonstrate the relationship between the centerline velocity and the dimensionless y-coordinate under different combinations of free and forced convection by changing the ratio of Grashof number and Reynolds number.

Figure 3 is the figure captured from Win Aung and G.Worku [9]. Figure 4 is the result obtained from ANSYS Fluent with the physical parameters listed in Table 1 and post-processed by MATLAB. Compare these two figures and it can be easily found out only little discrepancies exist which proves the accuracy of ANSYS Fluent under this condition. Only the independent variable is different which is caused by reversing x-coordinate and y-coordinate. From the figure, it shows that with zero value of the ratio of Grashof number versus Reynolds number, the centerline velocity is smooth, however, once the value of the ratio of Grashof number and Reynolds number increases, the distortion of the velocity profile becomes more significant. It is known that when the value of this ratio is equal to 0, it means that only forced convection is taken into consider and free convection is negligible. But as the increase of the value of this ratio, more and more free convection appears which lead to the distortion of the velocity profile. And it is obvious that the centerline velocity will approach to the same steady value finally only with a different rate. As more free convection included, it takes more time and that is why the entrance length becomes longer for the centerline velocity to reach the stable value. If the free convection is negligible, it can be realized by neglecting the gravity value directly in ANSYS Fluent. Otherwise, the gravitational acceleration should be input as -9.81m/s^2 to guarantee both forced and free convection exist.

Secondly, asymmetric wall temperature boundary condition is considered. Under this circumstance, the ratio of wall temperature difference is no longer one. Only the value 0.5 for this ratio is applied here to make a comparison with the figures listed in the reference paper, however, this ratio which is determined by the temperature between the cold and hot wall can be any value between 0 and 1. Once the value of this ratio changes, the velocity and temperature profile will be totally different. Given both the cold and hot wall temperature and input as the two boundary conditions into ANSYS Fluent, the software will automatically solve the problem and provide the reasonable and reliable solution. The relationship between the axial velocity and dimensionless x-coordinate at a fixed height under different combinations of free and forced convection is shown as the figure below respectively.

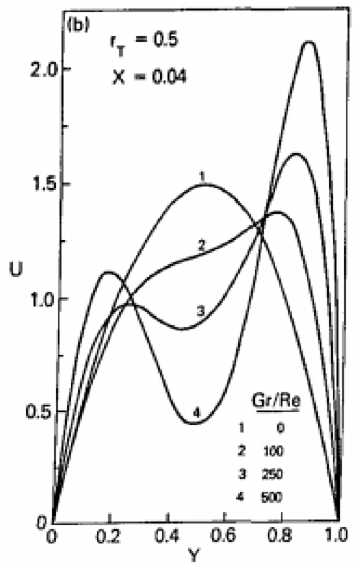

Fig. 5 Velocity distribution as a function of Gr/Re for asymmetric heating [9].

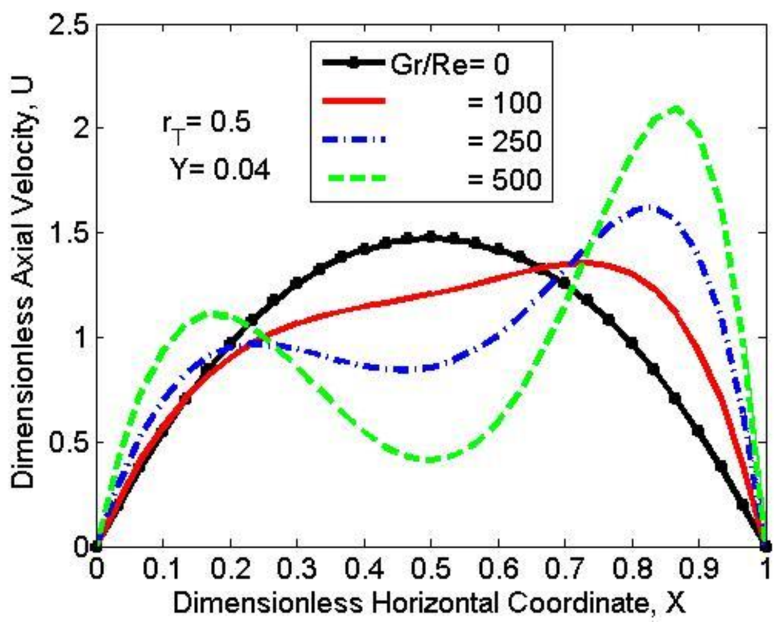

Fig. 6 Velocity distribution as a function of Gr/Re for asymmetric heating.

Make a contrast between Fig. 5 and Fig. 6, the developing trend for both figures are almost the same under the same condition with only little discrepancies existing. From Fig. 6, it can be easily found out that the red dashed line represents the axial velocity profile under the condition where only forced convection is considered. And if free convection is neglected and only forced convection is considered, then the relationship between the axial velocity and position will be a smooth hyperbola which is the same as Fig. 6 displays. As the increase of the value of the ratio of Grashof number over Reynolds number, the velocity profile will be distorted more obviously. The more value of this ratio, the more free convection is included in the convection problem. And it is obvious that if more free convection is included, the position to get the maximum velocity will be right shifted.

Lastly, only the velocity profile under either symmetric or asymmetric wall temperature boundary condition has been checked to be correct in the above paragraphs. However, another important parameter, temperature, should also be taken into consider to judge the accuracy of the model established in ANSYS Fluent. Here the temperature profile under asymmetric wall heating boundary condition is displayed in this paragraph. The existing result directly obtained from the reference paper is listed as Fig. 7 while ANSYS Fluent result under the same boundary condition is displayed in Fig. 8.

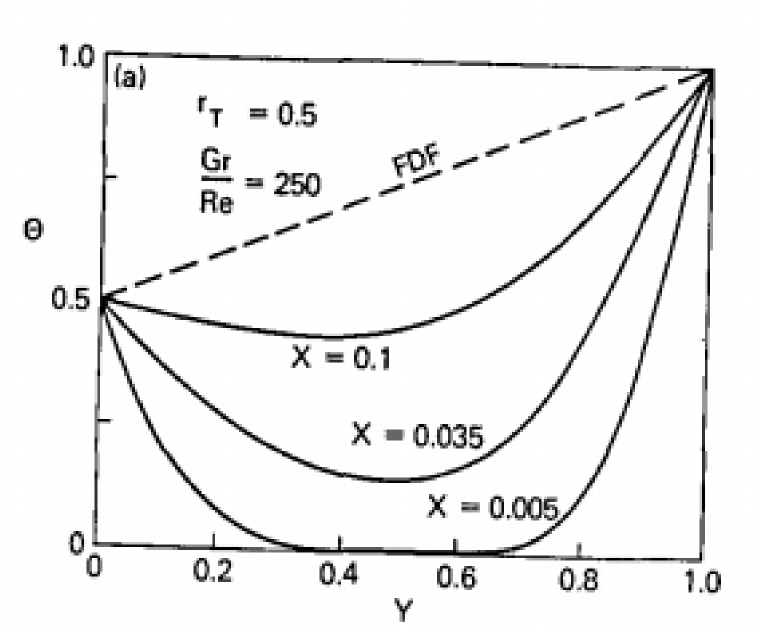

Fig. 7 Dimensionless temperature distribution under asymmetric heatng [9].

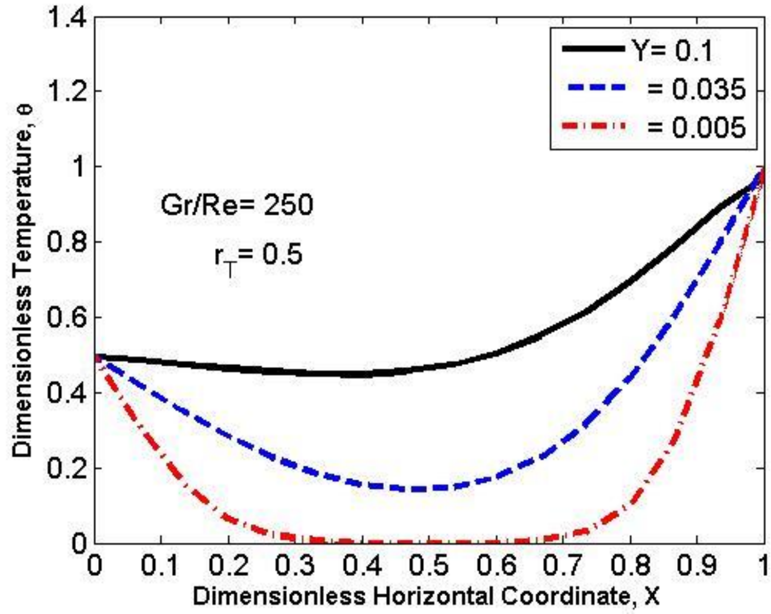

Fig. 8 Dimensionless temperature distribution under asymmetric heating.

Both Fig. 7 and Fig. 8 display the relationship between dimensionless temperature profile and dimensionless x-coordinate at different height respectively within the condition that both the Grashof number versus Reynolds number ratio and the temperature difference ratio r_T are kept constant. Compare Fig. 7 and Fig. 8 and it is apparent that the developing trend of the curve at the same height is almost the same. Both ratios are kept constant implying that the combination between forced and free convection are fixed and the boundary condition of both the cold wall and hot wall are kept constant as well. These two figures concentrate on how the temperature profile with respect to the x-coordinate will vary according to the change of the height of the plate under the fixed boundary condition. Both Fig. 7 and Fig. 8 show that as the height increases from bottom to top which means that the entrance length increases, the dimensionless temperature will be more easily to reach the equilibrium state.

In this section, both symmetric heating and asymmetric heating are checked. At the same time, both velocity and temperature profile under different conditions are compared to make a conclusion here. All the figures listed above in this sections prove that ANSYS Fluent provides a reliable and accurate solution for fluid flow between parallel plates under constant wall temperature boundary condition either symmetric heating or asymmetric heating. To make sure the model is precise enough for solving the problem of this paper, the model should also be checked under constant wall heat flux boundary condition in the following section.

2.2.2 Constant wall heat flux verification

Although this paper only focuses on the convection problem under constant wall temperature boundary condition, to make sure the established model is completely reliable, the fluid under constant wall heat flux boundary condition is also needed to be verified. If both thermal boundary conditions are checked to be reliable, then this model can be continuously utilized for prospective research. Another reference paper from Win Aung and G. Worku [10] could be seen as a standard reference for constant wall heat flux boundary condition. This reference paper employs the same material and same physical properties as the last reference paper in section 2.2.1. So, properties listed in Table 1 are still utilized in this section for verification. Velocity and temperature profile under either symmetric or asymmetric heat fluxes are analyzed respectively. The simulation results will be compared with the reference paper to make a conclusion. Firstly, mixed convection in ducts with symmetric wall heat flux is considered.

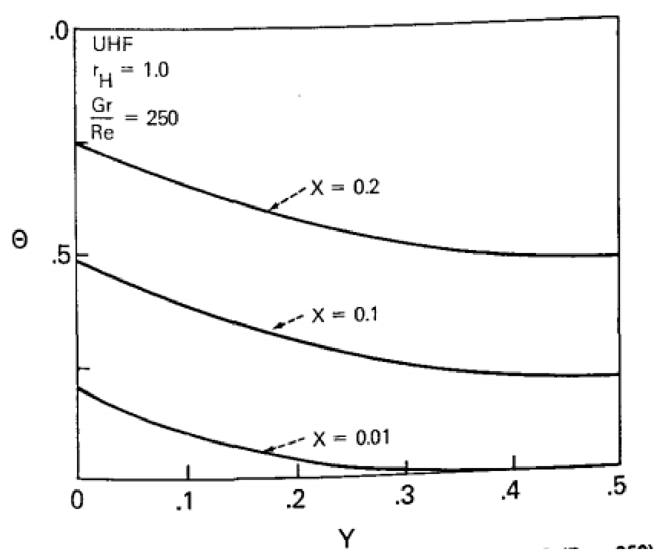

Fig. 9 Development of the temperature profile under symmetric heating [10].

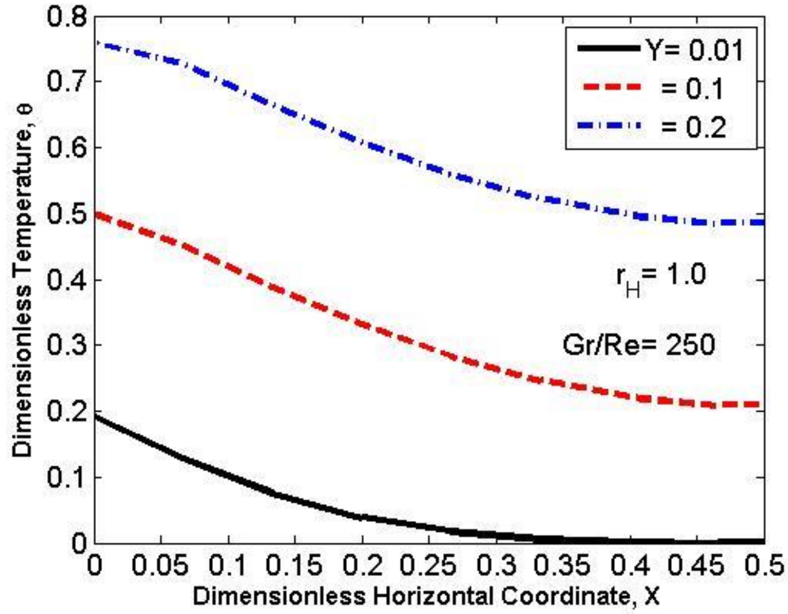

Fig. 10 Development of the temperature profile under symmetric heating.

Whether it is with symmetric wall heat flux or asymmetric wall heat flux is determined by a non-dimensional parameter r_H instead of r_T in section 2.2.1. r_H is the ratio of the heat flux from the two walls. Once this value is 1, it indicates that the fluid is under symmetric heat flux heating process. Otherwise, the fluid is under asymmetric heat flux heating.

For Fig. 9 and Fig. 10, the dimensionless parameter r_H is equal to 1 because it is under symmetric heating. Under symmetric heating, if the plot is for the full dimensionless x-coordinate, then the temperature profile versus dimensionless x-coordinate will be symmetric about the line dimensionless x-coordinate X= 0.5 with the same value of dimensionless temperature at the two boundaries and the minimum at the centerline. The value of Grashof number over Reynolds number is fixed to be 250 shows that the combination of free and forced convection is fixed. The only thing that matters the temperature profile over x-coordinate will be the height of the fluid in the duct. It is apparent that the developing trend of the dimensionless temperature versus dimensionless x-coordinate in Fig. 9 and Fig. 10 are almost the same for the fluid at the same height. The dimensionless temperature at the same x-coordinate will increase as the increase of the height. The fluid at a higher height means the fluid endures a longer entrance length before reaching that altitude. The longer the entrance length is, the less temperature difference between the wall and the interior fluid will be and the bigger value of the dimensionless temperature parameter will be.

Secondly, mixed convection in ducts with asymmetric heating is considered.

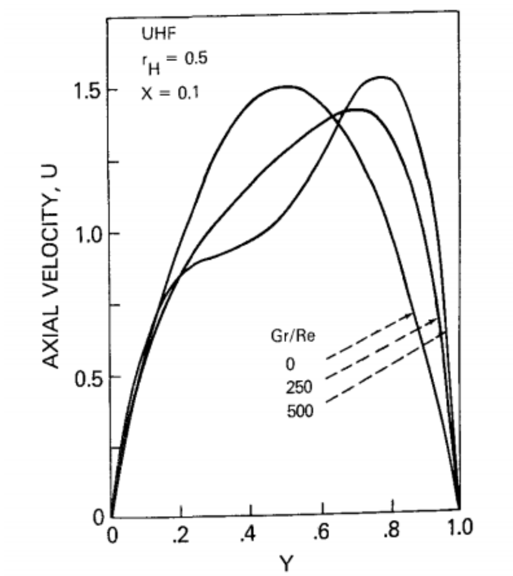

Fig. 11 Velocity profile under asymmetric heating [10].

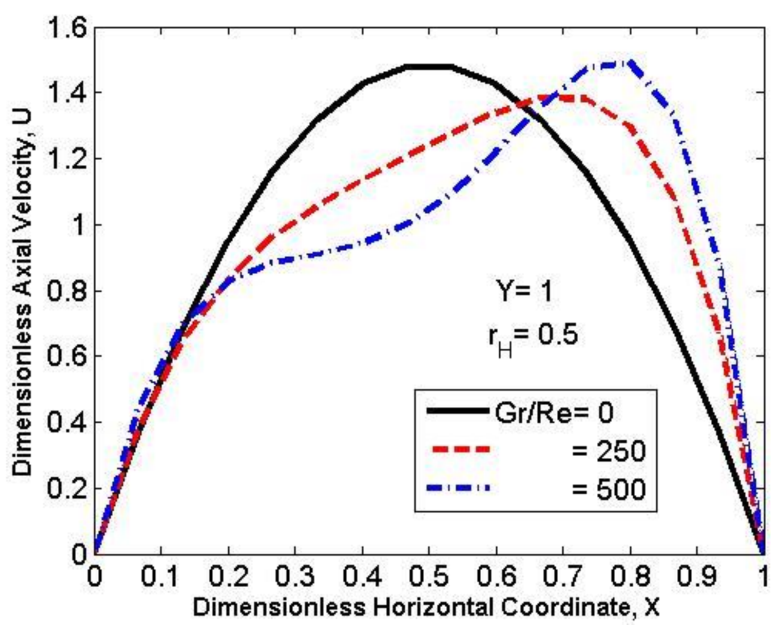

Fig. 12 Velocity profile under asymmetric heating.

The dimensionless parameter r_H will be a value between 0 and 1 instead of 1 under asymmetric heating. The value of this dimensionless parameter will be determined by the ratio of heat flux exerted on the two walls. The velocity and temperature profile will change totally once this dimensionless parameter changes. For this section, only the value of 0.5 is considered. The relationship between axial velocity and x-coordinate under the different combinations of free and forced convection at a same height in the duct is displayed in the figure below respectively.

Make a contrast between Fig. 11 and Fig. 12, the developing trend of velocity profile is approximately the same with the same Grashof number versus Reynolds number ratio. Once the value of Grashof number over Reynolds number is equal to 0, then the combined convection problem is transformed into pure forced convection problem. Under this circumstance, the velocity profile versus x-coordinate displays as a smooth hyperbola as imagined. If the value of Grashof number versus Reynolds number ratio increases, more temperature difference will exist between the hot wall and cold wall which will lead more free convection to form a combined convection with the existing forced convection. The wall at the right-hand side is assumed to be the hot wall or the wall with bigger heat flux value. The more free convection is included, the more significant the velocity profile will be distorted. In addition, the position to earn the maximum velocity will be right shifted starting from the centerline to the right-hand side with the increase of free convection.

In conclusion, based on all the comparisons between ANSYS Fluent results and existing results in literatures in section 2.2.1 and 2.2.2, the model established in ANSYS Fluent is accurate for at least Newtonian fluid under both two types of thermal boundary conditions including constant wall temperature boundary condition and constant wall heat flux boundary condition. For each type of thermal boundary condition, either velocity or temperature parameter under both symmetric heating and asymmetric heating has already been checked to be reliable. However, the mathematical model for our research object packer fluid is power-law fluid which is a kind of non-Newtonian fluid. So, the model should also be tested for non-Newtonian fluid to make sure it is suitable for the utilization of packer fluid. The following section will focus on the verification of non-Newtonian fluid.

2.3 Verification of non-Newtonian fluid

In section 2.2, air which is a kind of Newtonian fluid is selected to be the research object. The temperature and velocity profile for both kinds of thermal boundary conditions have already been verified to be reliable. So, the model established in ANSYS Fluent can be seen as reliable and accurate for Newtonian fluid. Here in this section, non-Newtonian fluid will be utilized to check the accuracy of the model.

As it is well known that the main difference between Newtonian fluid and non-Newtonian fluid is the viscosity. For Newtonian fluid, its shear stress and shear rate will perform a linear relationship, while for non-Newtonian fluid such as power-law fluid, its shear stress and shear rate will perform a more complicated nonlinear relationship. To transform the model from Newtonian fluid into non-Newtonian fluid, only the viscosity properties defined in material models should be changed. For Newtonian fluid, the viscosity should be kept the default constant setting. However, for power-law fluid, the properties should be transformed into non-newtonian-power-law setting. The power-law fluid is determined by two controlled parameters. One is flow consistency index K and the other is flow behavior index n. Newtonian fluid can be modeled as a special non-Newtonian fluid with the flow behavior index n equal to 1. Once all the other physical properties are kept constant as the ones set in section 2.2 with the only change of the model from constant viscosity fluid model to non-newtonian-power-law fluid model in the material properties setting part. ANSYS Fluent will use its default non-Newtonian fluid solver to solve the problem by simulation process. The results obtained here utilizing non-newtonian-power-law model after running the iterations will be totally the same as the results displayed in section 2.2 utilizing constant Newtonian fluid solver. To avoid repetition, the results will not be listed here again. So, non-newtonian-power-law model solver is also proved to be reliable. Based on all the verification processes stated above, the model established in ANSYS Fluent is verified to be reliable and it is applicable to solve the combined convection problem of packer fluid in the following sections.

3. Governing equations and formulas

The purpose of this paper is to solve laminar combined convection of packer fluid between two parallel plates. The flow is assumed to be two dimensional. All the physical properties are assumed to be constant except for the variation of density existing in the buoyancy force term of momentum equation. Assumptions of steady state and fully-developed flow are also made. The governing equations for solving this problem are listed in this section and the final solution is obtained by solving the problem numerically utilizing ANSYS Fluent.

The schematic diagram of this flow geometry is shown in the figure below. In this figure, *y*-direction is defined to be upwards and parallel to vertical plates, while horizontal direction is defined to be *x*-direction. At the same time, *u* is defined to be axial velocity in *y*-direction and *v* is defined to be transverse velocity in *x*-direction. Either constant wall temperature or constant heat flux boundary condition is utilized as thermal boundary condition. For this paper, only constant wall temperature boundary condition is considered.

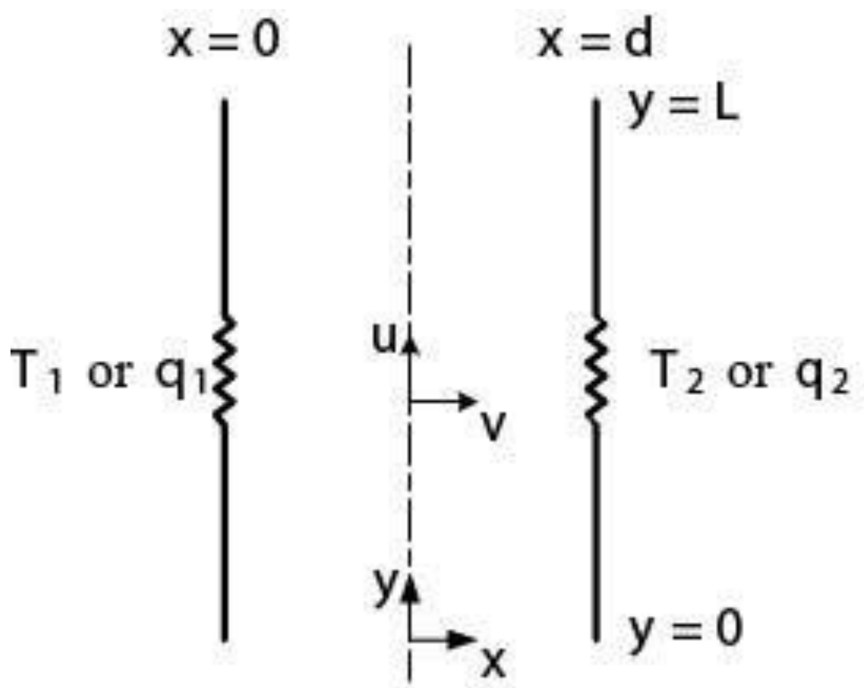

Fig. 13 Schematic diagram of the flow geometry.

The apparent viscosity for power-law fluid is defined as the formula below. All the parameters including viscosity term μ defined in this section will utilize the definition of apparent viscosity below.

$$\mu = K \left| \frac{\partial \mathrm{u}}{\partial x} \right|^{n-1} \tag{3.1}$$

where K and n are the two parameters for power-law fluid. K is the so-called flow consistency index and n is flow behavior index. For Newtonian fluid, n is equal to 1 and K is the constant viscosity under the specific fixed temperature. For this research object packer fluid, n is equal to 0.5 while K is equal to 0.09576. With the definition of apparent viscosity, the shear stress of the power-law fluid can be denoted by the product of apparent viscosity and shear rate.

$$\tau = \mu \left| \frac{\partial \mathrm{u}}{\partial x} \right| \tag{3.2}$$

Utilizing the definition of apparent viscosity, the governing equations including continuity equation, momentum equations and energy equation are listed as following respectively.

$$\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = 0 \tag{3.3}$$

$$\rho\left(u\frac{\partial u}{\partial y} + v\frac{\partial u}{\partial x}\right) = -\frac{\partial p}{\partial y} + \rho g \beta (T_1 - T_2) + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) \tag{3.4}$$

$$\rho\left(v\frac{\partial v}{\partial x} + u\frac{\partial v}{\partial y}\right) = -\frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) \tag{3.5}$$

$$\rho \mathrm{c}_p\left(u\frac{\partial T}{\partial y} + v\frac{\partial T}{\partial x}\right) = k\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) + \mu\left(\frac{\partial u}{\partial x}\right)^2 \tag{3.6}$$

The boundary conditions for this specific problem is defined as the formulas below.

$$at\, x = 0 : u = 0, v = 0, T = T_1 \tag{3.7}$$

$$\text{at}\, x = d : u = 0, v = 0, T = T_2 \tag{3.8}$$

$$\text{at}\, y = 0 : u = u_0, \mathrm{v} = 0, T = T_0 \tag{3.9}$$

The governing equations are nondimensionalized utilizing the dimensionless parameters and listed as below respectively.

$$\frac{\partial V}{\partial X} + \frac{\partial U}{\partial Y} = 0 \tag{3.10}$$

$$U\frac{\partial U}{\partial Y} + V\frac{\partial U}{\partial X} = -\frac{\partial P}{\partial Y} + \frac{Gr}{\mathrm{Re}}\theta + \frac{\partial}{\partial X}\left(\frac{\partial U}{\partial X}\right) \tag{3.11}$$

$$U\frac{\partial \theta}{\partial Y} + V\frac{\partial \theta}{\partial X} = \frac{1}{\mathrm{Pr}}\text{Ä}\frac{\partial^2 \theta}{\partial X^2} + \frac{1}{\mathrm{Re}^2}\frac{\partial^2 \theta}{\partial Y^2}) + E\mathrm{c}\left(\frac{\partial U}{\partial X}\right)^2 \tag{3.12}$$

The dimensionless parameters and dimensionless numbers included in the dimensionless form of governing equations are defined as following respectively.

$$U = \frac{u}{u_0}; V = \frac{\mathrm{v}d}{\nu} \tag{3.13}$$

$$X = \frac{x}{d}; Y = \frac{y}{d\,\mathrm{Re}} \tag{3.14}$$

$$P = \frac{p - p_0}{\rho {u_0}^2}; \theta = \frac{T - T_2}{T_1 - T_2} \tag{3.15}$$

$$\mathrm{Re} = \frac{\rho u_0 d}{\mu} \tag{3.16}$$

$$Gr = \frac{\rho^2 g \beta (T_1 - T_2) d^3}{\mu^2} \tag{3.17}$$

$$E\mathrm{c} = \frac{{u_0}^2}{c_p (T_1 - T_2)} \tag{3.18}$$

$$\Pr = \frac{c_p \mu}{k} \tag{3.19}$$

where v is kinematic viscosity which is defined by the quotient of apparent viscosity and density. Eckert number which is denoted by Ec is a dimensionless number which is related to viscous dissipation. However, for low speed flow, viscous dissipation can mostly be neglected.

When considering pure free convection problem, dimensionless parameter Rayleigh number is considered to determine the free convection intensity. The formula of Rayleigh number is defined as follow.

$$Ra = Gr \times \Pr \tag{3.20}$$

where Gr is Grashof number and Pr is Prandtl number defined as above.

When considering combined convection problem, dimensionless parameter Archimedes number is used to define the relative strength of free convection and forced convection and will be utilized to determine which type of convection is the dominant term in the following sections.

$$Ar = \frac{Gr}{\mathrm{Re}^2} = \frac{g\beta(T_1 - T_2)d}{u_0^{\ 2}} \tag{3.21}$$

where Gr is Grashof number and Re is Reynolds number defined in the formulas above.

Nusselt number which demonstrates the ratio of convective and conductive heat transfer needs to be analyzed for convection problems. How Nusselt number will vary with the key factors that influence the intensity of free convection and forced convection will be shown respectively in section 4. Nusselt number is defined as the formula below.

$$Nu = \frac{hD_h}{k} \tag{3.22}$$

where h is the convective heat transfer coefficient and D_h is hydraulic diameter. Convective heat transfer coefficient h is calculated based on the ANSYS Fluent data utilizing the formula defined below.

$$h = \frac{q_2}{T_m - T_2} \tag{3.23}$$

D_h is the so called hydraulic diameter. For rectangular parallel plate geometry, the hydraulic diameter is equal to two times the width of the duct. So, for this width fixed duct, hydraulic diameter is fixed to be 0.3ft.

$$D_h = 2 \times d \tag{3.24}$$

4. Convection problem of packer fluid

The motivation to analyze the convection problem of packer fluid is that the utilization of packer fluid can prevent or strongly reduce heat transfer from production fluid to the outer casing region in Oil & Gas Industry. In this chapter, pure free convection, pure forced convection and combined convection of this packer fluid are analyzed respectively in the following sections. How different parameters will affect the combined convection of this packer fluid will be shown respectively later in this chapter. The physical properties of this packer fluid that will be used in this section is displayed in Table 2 shown below.

Table 2 Physical properties of packer fluid

K	$0.09576 Pa \cdot s^n$
n	0.5
Density	$961.108 kg / m^3$
Thermal Conductivity	$0.17307 W / m \cdot K$
Specific Heat	$1800.324 J / kg \cdot K$
Thermal Expansion Coefficient	$0.0009 / K$

4.1 Pure free convection of packer fluid

4.1.1 Introduction of free convection

Before going to solve the combined convection problem of this packer fluid, free convection and forced convection of this packer fluid are analyzed respectively first. Free convection or natural convection is a kind of heat transport process in which the fluid motion is not generated by the external sources. Instead it is caused only by the density differences in the fluid due to temperature gradients. During free convection, the fluid which is adjacent to a heat source will be heated, become less dense and rise to a higher height because of density difference. At the same time, the cooler fluid will then move to replace the rising fluid. The cooler fluid is then heated and the process will continue which will cause convection current to transfer the heat energy from the bottom to the top. An axial velocity profile over the entire region under pure free convection will be exhibited to show the principle of free convection. The driving force of the free convection is the buoyancy force caused by the density differences.

4.1.2 Pure free convection of packer fluid

When studying free convection, the dimensionless number Rayleigh number should be considered. Rayleigh number determines the strength of free convection. The value of Rayleigh number is determined by the product of Grashof number and Prandtl number. When Rayleigh number exceeds some specific critical value, heat transfer is transferred primarily by convection. Otherwise, heat transfer is transferred mainly by conduction. How Rayleigh number is affected respectively by temperature difference and aspect ratio is displayed in Fig. 14. How Nusselt number will vary with Rayleigh number for laminar pure free convection is shown in Fig. 15.

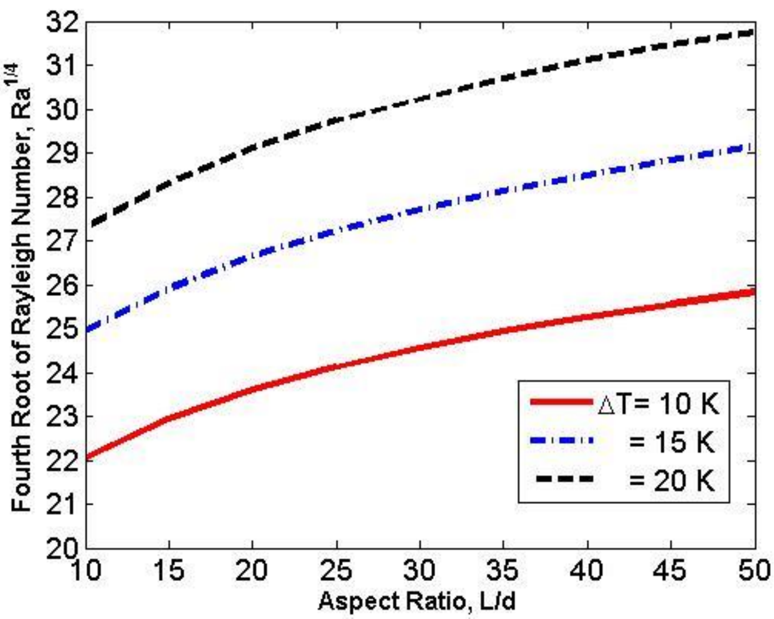

Fig. 14 Fourth root of Rayleigh number versus aspect ratio.

Fig. 14 shows the developing trend of Rayleigh number versus aspect ratio under pure free convection. The increase of the value of fourth root of Rayleigh number indicates the increase of Rayleigh number itself. The increase of aspect ratio is equivalent to the increase of the length of the duct by fixing the duct width. From each line in Fig. 14 respectively, Rayleigh number increases with the increase of aspect ratio. Compare the three lines with each other, Rayleigh number increases with the temperature difference at the same aspect ratio. The lines in the figure is distinguished by the temperature difference instead of the Grashof number because Grashof number is a parameter which is determined by the value of viscosity. Viscosity is not a fixed constant value with the change of the duct length. It lies between the range of 0.01 and 0.04 in according with different conditions. With the fixed specific heat and thermal conductivity value, Prandtl number which is only a function of viscosity varies in the range of hundred level according to the different value of viscosity under different conditions.

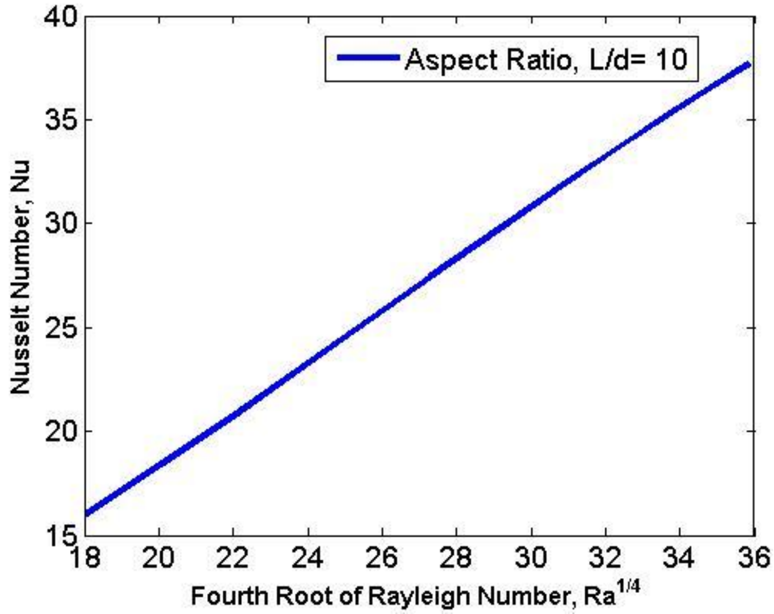

Fig. 15 Nusselt number versus fourth root of Rayleigh number for free convection.

Fig. 15 describes the relationship between Nusselt number and fourth root of Rayleigh number under pure free convection. From the figure, when Rayleigh number increases, Nusselt number will increase proportionally. It is obvious that the figure reveals linear relationship between Nusselt number and fourth root of Rayleigh number. This conclusion satisfies the conclusions made in existing literatures that for laminar free convection, Nusselt number will perform linear relationship with fourth root of Rayleigh number when Rayleigh number is kept in specific range, while for turbulent free convection, Nusselt number will perform linear relationship with cubic root of Rayleigh number. For this packer fluid, under pure free convection, Prandtl number lies in the range of thousand level and it varies with the variation of viscosity under different conditions.

Pure free convection of packer fluid is analyzed here. The following sections will discuss pure forced convection and combined convection of packer fluid respectively.

4.2 Pure forced convection of packer fluid

4.2.1 Introduction of forced convection

In this section, pure forced convection of this packer fluid is studied. Forced convection which is totally different from free convection described in the above section is a kind of mechanism in which the fluid motion is generated by an external source such as a pump or a fan. Forced convection is regarded as one of the main efficient ways to transfer heat energy. In daily life, forced convection has broader applications than free convection and can be encountered more often in everyday life. Central heating, air conditioning, steam turbines and heat exchangers are the typical mechanisms where heat is mainly transferred by forced convection.

4.2.2 Display of pure forced convection of packer fluid

As described above, pure free convection which is generated only by the density difference caused by temperature difference has no relationship with external sources. The velocity input at the inlet is set to be zero for pure free convection in ANSYS Fluent. However, for forced convection, because of the existence of the external sources such as a fan or a pump, the fluid will have an exact value of input velocity at the inlet. The magnitude of the fluid axial velocity at any position will be determined by and proportional to the inlet velocity. To study pure forced convection in ANSYS Fluent, free convection must be neglected through setting gravitational acceleration to be zero in setup procedure. The figure below is the dimensionless outlet velocity profile under the condition of a 0.05m/s inlet velocity.

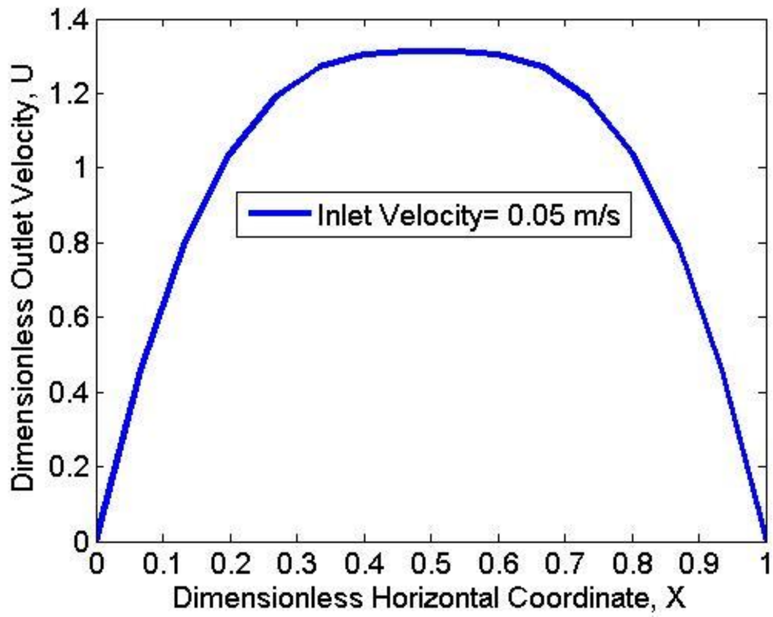

Fig. 16 Dimensionless outlet velocity for pure forced convection.

Fig. 16 provides the relationship between outlet dimensionless velocity and dimensionless x-coordinate. The velocity profile is independent of the length of the pipe in this enlarged region which illustrates that the fluid has already lied in fully developed region. Combine the velocity values exhibited on the left-hand side, and it is easy to find out that the velocity gets its maximum around the centerline while the minimum is acquired at the boundaries because of no-slip boundary condition.

Fig. 16 shows the dimensionless outlet velocity profile. The velocity profile performs like a smooth hyperbola as expected. The influential factor for the intensity of forced convection is the magnitude of inlet velocity. The inlet velocity can be associated with Reynolds number according to the viscosity under different conditions. The relationship between Nusselt number and Reynolds number of this packer fluid for pure forced convection is displayed in the figure below.

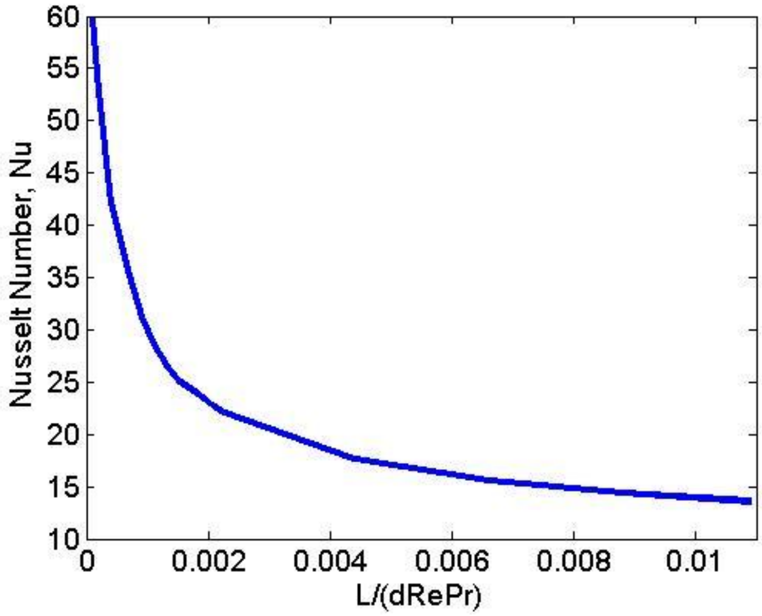

Fig. 17 Nusselt number for pure forced convection.

Fig. 17 demonstrates how Nusselt number will vary with the combination effects of aspect ratio, Reynolds number and Prandtl number. On the denominator of the independent variable, the viscosity of Reynolds number and Prandtl number cancel with each other which makes the denominator a constant because all the physical properties are kept constant for the packer fluid. So, Fig. 17 actually shows how Nusselt number will vary with the relationship of aspect ratio for pure forced convection. From the figure, it is obvious that when aspect ratio increases, Nusselt number will decrease with a decreasing rate. As aspect ratio continues to increase to a larger value, Nusselt number will keep decreasing and approaching to a steady value at last.

After analyzing the pure free and forced convection respectively in the above two sections 4.1 and 4.2, combined free and forced convection problem of this packer fluid is solved in the following section 4.3.

4.3 Combined convection of packer fluid

4.3.1 Introduction of combined convection

During forced convection processes, some amount of free convection is always present whenever the gravitational force exists. When the natural or free convection cannot be seen as negligible, such flows are typically identified as under mixed convection. If both forced and free convection are needed to be considered rather than one type of convection is neglected when compared with the other type, then combined convection problem should be solved. When considering both free convection and forced convection at the same time, the intensity of these two types of convection should lie in a comparable region. There is a dimensionless parameter called Archimedes number which is a judgment of the relative strength of free and forced convection. The value of this parameter is determined by the ratio of Grashof number over the square of Reynolds number. Once this value is around one, both free and forced convection should be considered because the magnitude of both types of convection is in the same level. Once this value is far more than 1, the natural convection or free convection will dominate because of the existence of huge temperature difference between the two walls. If this parameter is far less than 1, then Reynolds number which is a representative of velocity parameter is much larger when compared with Grashof number which represents temperature difference. This condition indicates that the fluid will have a relatively large inlet velocity which is caused by forced convection. If one type of convection is negligible by calculating this Archimedes number, then the solution obtained by considering only the other kind of convection is accurate. However, ANSYS Fluent will directly calculate the reliable result with both free and forced convection considered according to the program embedded into the software. In the following paragraphs, how different factors will influence combined convection will be studied and shown respectively, and conclusions will be made based on comparison between ANSYS Fluent result figures.

4.3.2 Combined convection of packer fluid

Because the problem analyzed here is combined convection of this packer fluid, so both the influential factors of forced convection and free convection can have some effects on the combined convection. How combined convection will be affected by aspect ratio; inlet velocity and temperature difference is analyzed respectively in the following paragraphs. When the relationship between one factor and combined convection is studied, the other factors should be kept fixed. For combined convection problem, the gravitational acceleration g should be set with a value of $-9.8 m/s^2$ in ANSYS Fluent setup step to make sure free convection is considered. At the same time, the fluid should be set with a nonzero inlet velocity according to the intensity of forced convection in order to include forced convection. Nusselt number which describes the ratio of convective to conductive heat transfer is a judgment of strength for combined convection.

4.3.2.1 Nusselt number versus aspect ratio

Firstly, how aspect ratio influences the combined convection of packer fluid is analyzed. Under this circumstance, the other two influential factors including the temperature difference between the hot wall and the cold wall and the inlet velocity are both fixed. How Nusselt number will vary with the change of aspect ratio is shown in the figure below. The inlet velocity is fixed to be 0.1m/s while the temperature difference between the two walls is set to be 10K and 20K respectively. If a too large temperature difference is set, then free convection will dominate and there will be reverse flow near the cold wall. And if a too large inlet velocity is set as an input, then forced convection is assumed to be the dominant term. So, these parameters should be set in a proper region in case that one type of convection is negligible when compared with the other type of convection. Using the velocity and temperature difference parameters to check Archimedes number defined in section 4.3.1, Archimedes number is found to be 0.4037 and 0.8074 for temperature difference equal to 10K and 20K respectively. Because Archimedes number is neither far more nor far less than 1, both free and forced convection should be taken into account which means that the problem is a combined convection problem.

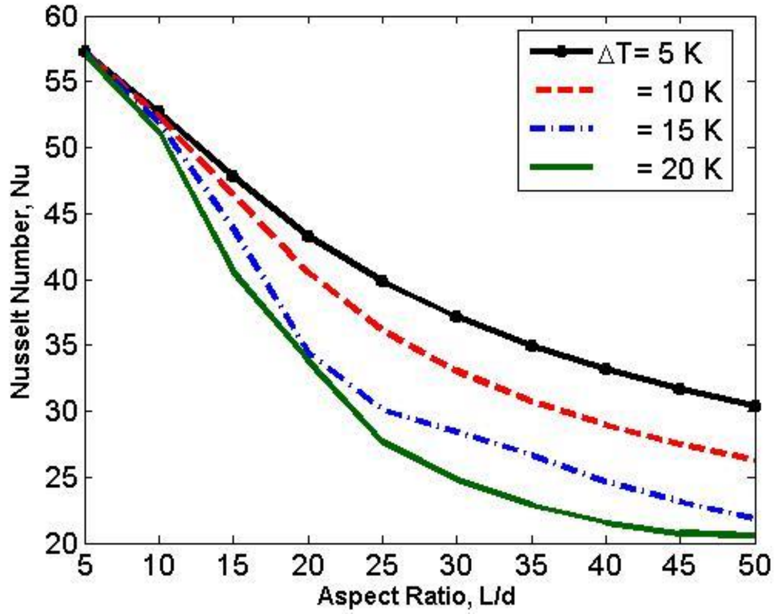

Fig. 18 Nusselt number versus aspect ratio for combined convection.

Fig. 18 shows Nusselt number of this packer fluid versus aspect ratio under temperature difference of 5K, 10K, 15K and 20K respectively with a fixed 0.1m/s inlet velocity. Fig. 18 demonstrates that at one fixed temperature difference and one fixed inlet velocity, Nusselt number will decrease with the increase of aspect ratio. If the length of the ducts increases, Nusselt number will decrease. After the length of the ducts has reached some value under this condition, Nusselt number will approach a steady value which means that the convection becomes stable and the fluid lies in fully developed region. Compare the four lines with the same aspect ratio in Fig. 18, and it is obvious that Nusselt number of the fluid within lower temperature difference is larger than Nusselt number of the fluid within higher temperature difference. Prandtl number is almost constant for this condition which lies in a range between 200 and 250. The figure is distinguished and denoted by existing temperature difference instead of Grashof number for the reason that Grashof number is not a constant. Grashof number is influenced by the value of viscosity which is not a constant even under the same temperature difference. The viscosity will change slightly within the condition of different aspect ratio value. How temperature difference will have impacts on Nusselt number will be shown later in the following paragraph.

4.3.2.2 Nusselt number versus temperature difference

Secondly, how the existence of temperature difference between the hot wall and cold wall affects Nusselt number is analyzed. Under this circumstance, with the change of value of temperature difference, both inlet velocity and aspect ratio should be kept constant. The duct has a 5ft length and 0.15ft width in dimension which means that a duct with an aspect ratio of 33.3333 is utilized.

Described as above, the value of temperature difference will mainly affect the strength of free convection. At the beginning, when temperature difference is small, there may not be reverse flow detected. However, as the temperature difference increases to some large value, ANSYS Fluent will detect reverse flow near the cold wall. To make an equilibrium between two types of convection, the intensity of forced convection should also be enhanced which means that the fluid should be given a larger velocity at the inlet. How Nusselt number will vary with the change of temperature difference under different velocities are shown in the figure below.

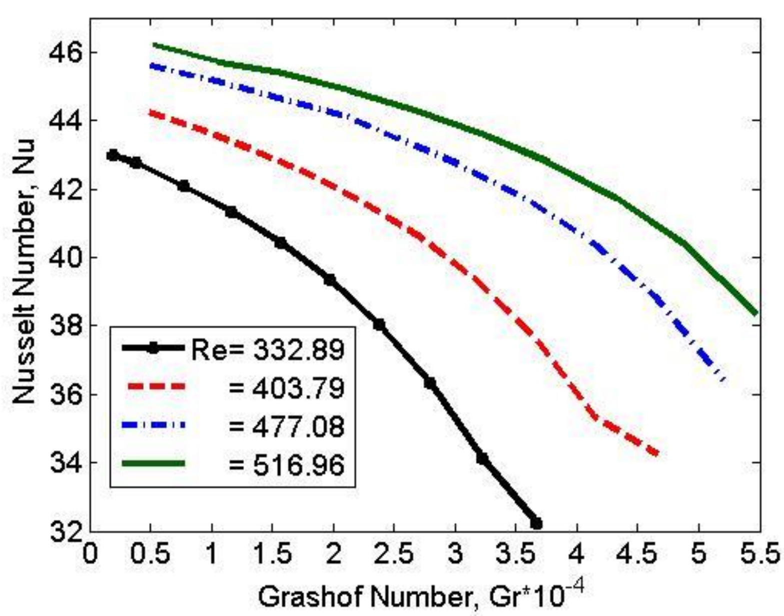

Fig. 19 Nusselt number versus Grashof number for combined convection.

Under the same inlet velocity, the viscosity varies very slightly and it is almost a constant in this small temperature variation range. For this reason, Reynolds number of each inlet velocity is given and calculated based on the average viscosity. Prandtl number is approximated to be around 200 for the packer fluid under these conditions. However, under different inlet velocities, the value of viscosity lies in different range even under the same temperature difference. So even under the same value of Grashof number does not mean the packer fluid is under the same temperature difference because of the differences between viscosities under different inlet velocities. From Fig. 19, it shows that even under the same temperature difference range, Grashof number will lie in different regions. For the same range of temperature difference, the range of Grashof number varies from each other which demonstrates the variance of viscosity under different inlet velocities. For a larger inlet velocity, Grashof number lies in a higher range of value which means viscosity decreases with the increase of inlet velocity. Viscosity varies slightly within a small temperature variance while it varies tremendously with the change of magnitude of inlet velocity.

Fig. 19 gives a view of how Nusselt number will vary with Grashof number within different inlet velocity conditions. All the four lines in Fig. 19 show a decreasing trend of Nusselt number with the increase of temperature difference or Grashof number. This satisfies the conclusion made in section 4.3.2.1 that Nusselt number will decrease with the increase of temperature difference. However, when making a contrast between the four lines with each other, it is obvious that at the same value of Grashof number, Nusselt number increases with the increase value of Reynolds number or the inlet velocity. How Nusselt number is affected by inlet velocity or Reynolds number will be studied in the following section in details. Keep increasing temperature difference, free convection will become dominant and reverse flow near the cold wall will become detected gradually at higher temperature difference. To make an equilibrium between free and forced convection, inlet velocity should also be enhanced to make more forced convection included.

How Nusselt number will vary with Reynolds number will be shown in the following section. The conclusions made in section 4.3.2.1 and 4.3.2.2 can also be verified in the following paragraphs through comparison.

4.3.2.3 Nusselt number versus Reynolds number

The relationship between Nusselt number and Reynolds number is analyzed here in this section. When studying on the effects of inlet velocity, both temperature difference and aspect ratio should be kept constant. The temperature difference is fixed to be 5K, 10K, 15K and 20 K respectively while the length of the duct is fixed to be either 5ft or 10ft. How Nusselt number will vary with the increase of inlet velocity under combinations of fixed aspect ratio and temperature difference is analyzed respectively in the figures listed below. Firstly, the fluid flowing under different temperature differences is analyzed respectively within a 5ft length duct. Following is the figure of Nusselt number versus Reynolds number within a 10ft length duct. A comparison between these two duct lengths under the same temperature difference 5K and 10K is listed at last to make conclusions how different factors affect Nusselt number.

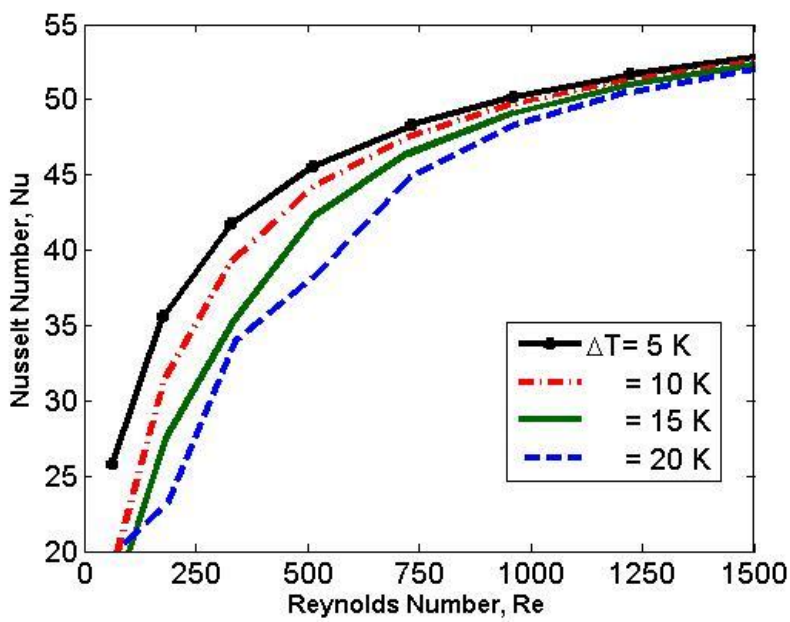

Fig. 20 Nusselt number versus Reynolds number within 5ft length duct.

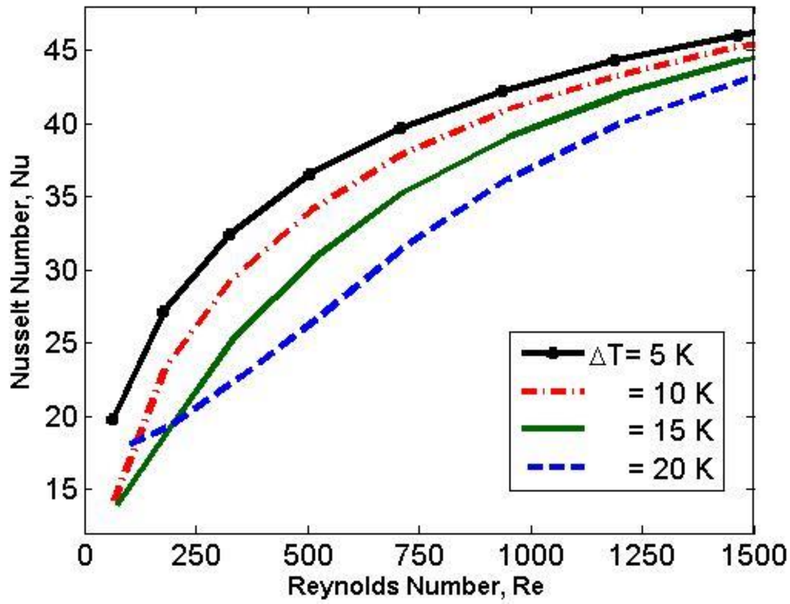

Fig. 21 Nusselt number versus Reynolds number within 10ft length duct.

Fig. 20 and Fig. 21 show how Nusselt number will vary with Reynolds number within 5ft and 10ft length duct respectively. All the four lines in both figures demonstrate that once Reynolds number or inlet velocity increases, Nusselt number will increase accordingly. When comparing among the four lines in each figure, it is obvious that for a fixed aspect ratio and for a fixed inlet velocity, the value of Nusselt number will decrease with the increase of temperature which satisfies the conclusion made in section 4.3.2.2. When analyzing each line respectively, under a fixed temperature difference, as the value of inlet velocity increases, Nusselt number will increase. There will not be reverse flow detected because forced convection approaches to become the domination term as inlet velocity increases. Check Archimedes number here and the value of Archimedes number will decrease accordingly with the increase of inlet velocity. As forced convection becomes dominant, Archimedes number will become far less than 1. And the solution obtained by neglecting free convection and considering forced convection only is accurate because the intensity of free convection is negligibly small when compared with forced convection. For these two figures, the lines are distinguished with different temperature differences instead of Grashof number because Grashof number which is determined by viscosity is not a constant value even within the condition of same temperature difference. The viscosity varies a lot under different temperatures which lead to a difference of Prandtl number as well. Prandtl number will be within hundred level varying from 100 to 350 according to viscosity difference under different conditions.

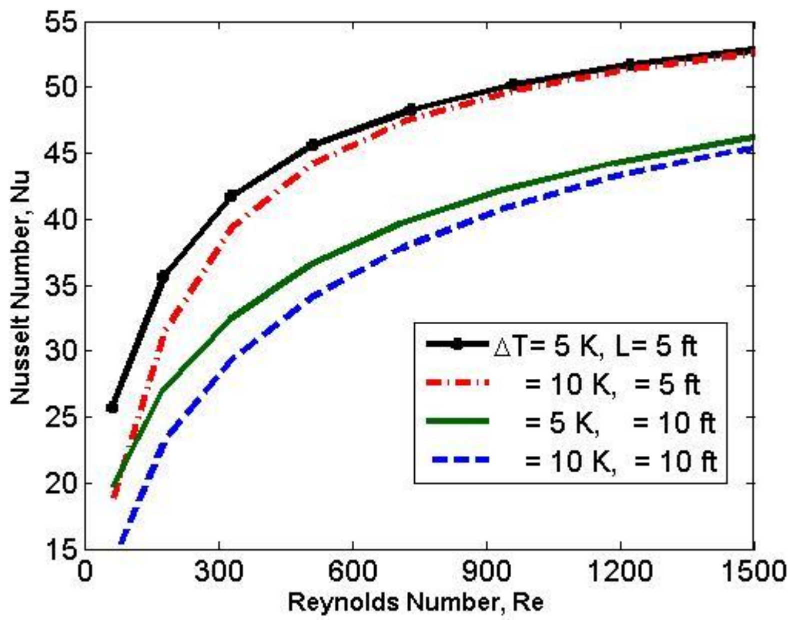

Fig. 22 Nusselt number versus Reynolds number comparison.

Fig. 22 describes how Nusselt number will vary with Reynolds number under the same temperature difference but within different length duct or within the same duct but under different temperature difference. As shown in the figure, the black line and the green line represent Nusselt number under 5K temperature difference while the red line and the blue line represent Nusselt number under 10K temperature difference. The black line and the red line represent the packer fluid flowing within a duct of 5ft length while the green line and the blue line represent the packer fluid flowing within a duct of a longer duct with 10ft length. When comparing the two lines under the same temperature difference, at the same value of Reynolds number, Nusselt number will decrease with the increase of the duct length or the aspect ratio by fixing duct width. This conclusion satisfies the conclusion made in section 4.3.2.1. When comparing the two lines within the same length of duct, at the same value of Reynolds number, Nusselt number will decrease with the increase of existing temperature difference between hot wall and cold wall which satisfies the conclusion made in section 4.3.2.2. Same Reynolds number does not mean that the same inlet velocity because of the difference of viscosity under different conditions. The viscosity will vary tremendously with the increase of velocity while the viscosity will vary slightly within the small temperature difference range. The main effect that determines the magnitude of viscosity is inlet velocity which performs a decreasing trend of viscosity with the increase of inlet velocity. The change of viscosity will affect the dimensionless parameters including Reynolds number, Grashof number, Prandtl number and Rayleigh number as well.

In the following section, how Nusselt number for combined convection will vary with the change of Archimedes number under different conditions is illustrated.

4.4 Nusselt number versus Archimedes number

When considering combined convection problem, Archimedes number should be checked. Whether both type of convection should be considered or one is small comparable to the other is determined by this dimensionless parameter. When Archimedes is neither too larger than 1 nor too smaller than 1, both type of convection should be considered. Fig. 23 shows how Nusselt number will vary with Archimedes number in the comparable region of both types of convection.

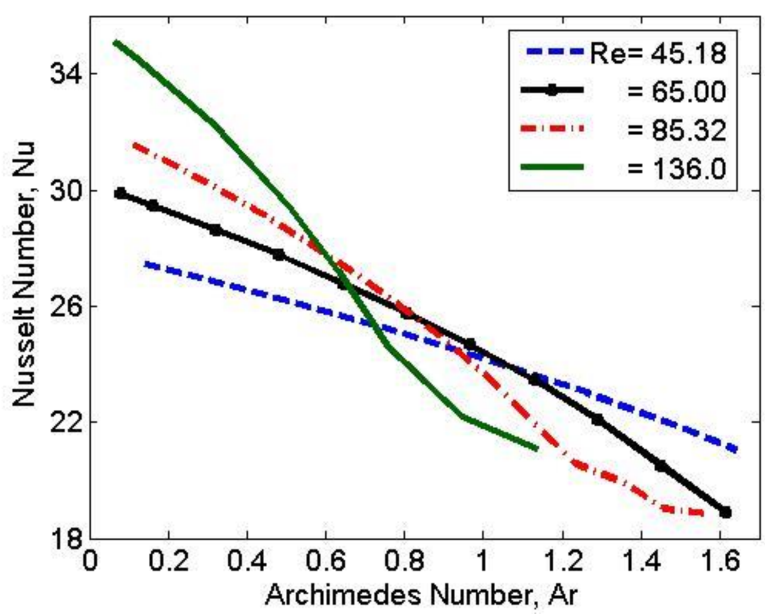

Fig. 23 Nusselt number versus Archimedes number for combined convection.

The figure only focuses on the region where Archimedes number is around 1 so that both types of convection should be considered. Fig. 23 analyzes the relationship between Nusselt number and Archimedes number by fixing temperature difference and changing inlet velocity or Reynolds number. Each line in Fig. 23 indicates that Nusselt number will decrease with the increase of the value of Archimedes number. As Reynolds number increases, the decreasing range of Nusselt number within the same range of Archimedes number becomes larger which leads to the increase of decreasing rate of Nusselt number under high Reynolds number. For this reason, Nusselt number of lower speed fluid will be more steady when compared with high speed fluid. Archimedes number which is the quotient of Grashof number and the square of Reynolds number is determined both by these two dimensionless parameters. Either the decrease of Reynolds number or the increase of Grashof number will lead the increase of Archimedes number. The decrease of Reynolds number will lead to the decrease of Nusselt number which satisfies the conclusion made in section 4.3.2.3. The increase of Grashof number will lead to the decrease of Nusselt number which satisfies the conclusion made in section 4.3.2.2.

After analyzing the combined convection problem of this packer fluid, how Nusselt number will vary with two power law fluid indexes K and n is shown respectively. This parameter analysis can have positive meanings for prospective research. According to the figures listed below, when solving combined convection of power law fluid model with different parameters, it is more straightforward to make a quick guess in which region its Nusselt number will lie under different conditions. The figure is obtained by keeping the other physical properties fixed and only changing the power law consistency and behavior index respectively. Only Nusselt number over Archimedes number is shown here, because the viscosity term will be cancelled in the calculation of Archimedes number. Otherwise, with the change of these two power law indexes, the viscosity of the fluid will be totally different. Once the viscosity is different, dimensionless parameters such as Reynolds number, Prandtl number, Grashof number and Rayleigh number will lie in totally different range. And under the same temperature difference or the same inlet velocity, Nusselt number over either Grashof number or Reynolds number for different power law fluid parameters will lie in totally different regions. So Nusselt number over

Archimedes number is shown respectively with the change of two power law indexes respectively.

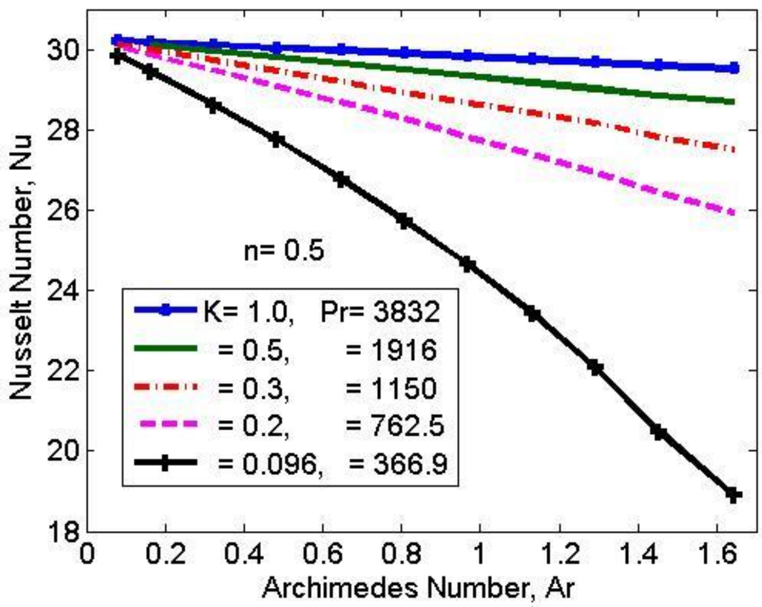

Fig. 24 Nusselt number versus Archimedes number at different K value.

Fig. 24 shows the developing trend of how Nusselt number will vary with the change of Archimedes number with different power law consistency index *K*. During the change of consistency index *K*, the behavior index of power law fluid is fixed to be 0.5. From each line in Fig. 24, it is easy to find out that no matter what value *K* is, Nusselt number will decrease with the increase of Archimedes number just as Fig. 23 displays. However, as *K* increases, the range of variation for Nusselt number becomes smaller and the decreasing speed of Nusselt number versus Archimedes number becomes lower. When *K* is around 0.1, the variation range of Nusselt number is almost 12. When the value of *K* is increased to 0.2, the variation range is approximate 4 while when *K* is increased to either value of 0.3, 0.5 and 1, the whole variation range of Nusselt number is no more than 2. If *K* is larger, Nusselt number for combined convection becomes more stable with the change of Archimedes number. And from the figure, when Archimedes number is almost 0, the value of Nusselt number seems to converge to a stable point where Nusselt number is independent of the value of consistency index *K*. This illustrates that in the range where forced convection becomes the domination term, Nusselt number is almost a constant value. Each line is obtained by changing the temperature difference within the same inlet velocity. Under this circumstance, the viscosity varies slightly and Prandtl number can be obtained by the average value of viscosity under different values of *K*. When *n* is fixed, Prandtl number of the fluid is proportional to the consistency index *K*. For the packer fluid within this condition, Prandtl number is around 366 while for *K* equal to 1, Prandtl number is around 3832. This figure will provide a good guess for researchers considering combined convection of power law fluid with behavior index *n*=0.5 with different consistency index *K*. According to the value of behavior index *K*, approximate profile of Nusselt number versus Archimedes number can be obtained. After analyzing the influential effects of consistency index *K* on Nusselt number, how Nusselt number will be affected by another power law parameter behavior index *n* is shown in the figure below.

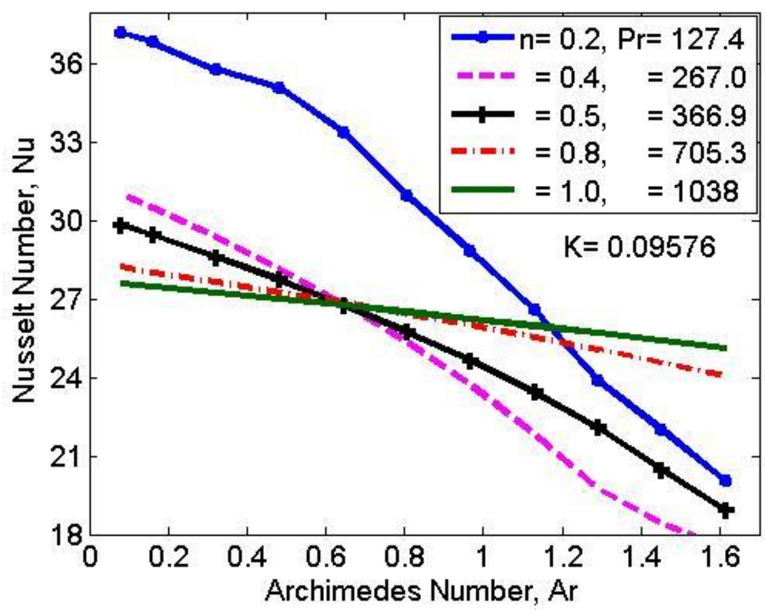

Fig. 25 Nusselt number versus Archimedes number at different n value.

Fig. 25 shows the relationship between Nusselt number and Archimedes number with different values of power law behavior index *n*. The power law consistency behavior *K* is defined to be constant. Judging from each line, no matter what value *n* is, Nusselt number will perform a decreasing trend with the increase of Archimedes number. Compare the five lines with each other, it is obvious that the variation range of Nusselt number decreases with the increase of power law behavior index *n*. As the increase of the power law behavior index *n*, the decreasing rate of Nusselt number becomes lower and Nusselt number becomes more stable with the variation of Archimedes number. When Archimedes number is approximately 0 which means that forced convection is the domination term when compared with free convection term, Nusselt number decreases with the increase of the behavior index *n*. It is reasonable to make a conclusion here for power law fluid under pure forced convection, Nusselt number decreases with the increase of power law behavior index *n*. For each line itself, Archimedes number varies by keeping the inlet velocity constant and changing temperature difference step by step. Viscosity varies slightly with the fixed inlet velocity in a small temperature variation range, so Prandtl number can be calculated based on the average value of viscosity. When consistency index *K* is fixed, viscosity or Prandtl number varies a lot with the change of the behavior index *n*. This relationship between the viscosity under different behavior index is not a simple linear relationship as consistency *K*. For *n*= 0.2, Prandtl number is around 127. For *n*= 0.4, Prandl number is around 267. For *n*= 0.5, Prandtl number is around 367. For *n*= 0.8, Prandtl number is around 705. While for *n*= 1, Prandtl number is a constant around 1038 because the fluid becomes Newtonian fluid and the viscosity becomes a constant instead of a variable dependent on shear rate. This figure can provide a view of how Nusselt number will vary with Archimedes number under different power law behavior index *n* and give researchers a good guess for their Nusselt number when analyzing combined convection problem of power law fluid with a consistency index approximate to 0.1. When a definite *n* value is given and *K* is around 0.1, approximate profile of Nusselt number versus Archimedes number can be obtained.

4.5 Display of velocity profile for combined convection

In this section, the dimensionless axial velocity profiles of this packer fluid at different heights under combined convection are shown respectively. For this packer fluid, the consistency index *K* is fixed to be 0.09576 while the behavior index n is 0.5. Velocity profiles under different Archimedes number is shown to analyze how different combinations of free and forced convection will have effects on the velocity profile during the developing region. Archimedes number is chosen to be 0.4, 0.8, 1.2 and 1.6 respectively which is neither far more nor far less than 1 so that both forced and free convection should be taken into consider. From fig. 26 shown below, it is obvious that as the increase of aspect ratio, velocity profile approaches towards fully developed region.

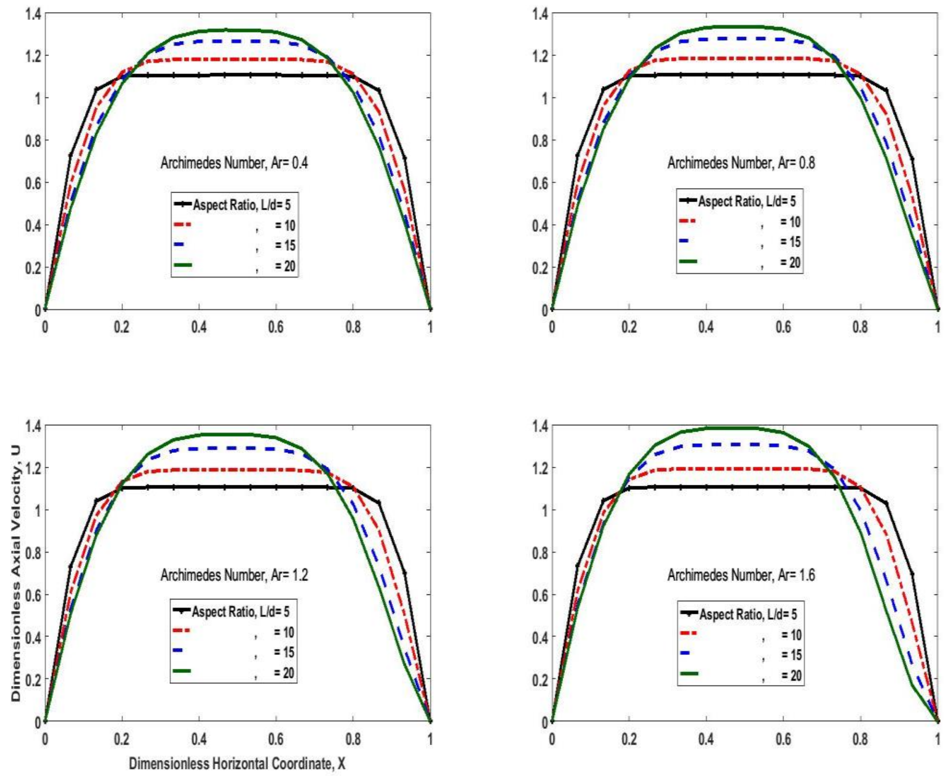

Fig. 26 Dimensionless axial velocity for combined convection

5. Conclusions

Firstly, pure free convection which is generated only by density differences due to temperature gradients instead of external source is analyzed in section 4.1. Rayleigh number which determines the strength of free convection increases with the increase of aspect ratio and the increase of temperature difference. So, make a conclusion here, natural or free convection is more rapid within a condition of a larger temperature difference and a larger distance through the whole convection region.

Secondly, more commonly utilized mechanism forced convection is analyzed in section 4.2. Forced convection is enhanced by increasing the magnitude of inlet velocity. Output velocity profile under pure forced convection displays like a smooth hyperbola as imagined. However, during the process of forced convection, free convection always exists because of the existence of gravitational force. For this reason, combined convection problem is needed to be solved.

Finally, based on analysis of combined convection problem of packer fluid in section 4.3 and 4.4, conclusions about how different influential factors will have effects on combined convection are made here. When considering combined convection problem, Archimedes number is always needed to be checked to determine whether both types of convection should be taken into consideration or one type of convection is the dominant term. If Archimedes number is far less than 1, forced convection will become dominant. If Archimedes number is far more than 1, free convection will play a leading role. If one type of convection is negligible when compared with the other type, then solution considering individually the dominant type of convection is reliable and accurate. Otherwise, if Archimedes number lies between 0 and 1, the intensity of free and forced convection will be in a comparable region and both free and forced convection will need to be considered.

The intensity of combined convection is determined by free convection and forced convection simultaneously, so both influential factors that have effects on either forced convection or free convection will have effects on combined convection. How Nusselt number will vary with aspect ratio, Grashof number and Reynolds number in specific range is analyzed respectively by fixing the other two parameters. Under the condition of fixed temperature difference and inlet velocity, Nusselt number will decrease with the increase of duct length or aspect ratio by fixing the duct width. Under the condition of fixed aspect ratio and inlet velocity, Nusselt number will decrease with the increase of the temperature difference or Grashof number. Under the condition of fixed aspect ratio and temperature difference, Nusselt number will increase with the increase of inlet velocity. The increase of Nusselt number will enhance the intensity of convection. So, to avoid convection from production fluid to outer casing annuli region as much as possible, the value of inlet velocity should be kept as small as possible while the length of the duct should be kept as large as possible. Finally, how the behavior index *n* and consistency index *K* will influence combined convection of power law fluid are analyzed respectively for prospective research.

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